Physics 1 section 7 newsletter...26 Jan 76...Mark Zimmermann, 133 Bridge, X1665
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I'd like to apologize for my lousy performance Wednesday...my only excuse is that I got up at 5:30 am that day for a T'ai Chi Ch'uan class...and since I'm going to be doing that for the rest of the term, maybe it would be best if you'd all skip the Wed. session....

About the quiz, first the answers: a) $M^2=2m(m+E_+)$, b) $E_{det}=m$, c) $E_{un}=E_+$, d) $\sin\theta=m/E_+$. I'll bet this was a pretty tough quiz for most people, though if you just stayed calm & conserved E & $\vec{p}$ and recalled that $E^2=p^2+m^2$, you got it all via a little algebra. After grinding it out, I saw how simple-looking the answers were for b),c),& d), so maybe there's a clever trick whereby you can leap to the answer at once...but I don't see it yet. The orthodox route goes as follows:

a) Before interaction, total energy is $E_+ + m$, the sum of the total energies of positron & e^-. Total momentum before is $\vec{p}_+ + 0$, sum of the momenta of e^+ & e^-; I'll define the $+\vec{x}$ direction to lie along the e^+'s motion, and the $+\vec{y}$ to be along the detected photon's velocity vector. Then, using $p^2=E^2-m^2$, we find that $\vec{p}_+=\mathrm{SQRT}(E_+^2-m^2)\hat{i}$. Conservation of E & $\vec{p}$ means that the positronium must have the same total E & $\vec{p}$, and thus that the positronium mass $M^2=E_{tot}^2-p_{tot}^2 =2E_+m+2m^2$, in units where c=1.

b) Now we have to be careful about <u>direction</u> in conserving the <u>vector</u> $\vec{p}$; we conserve E, p_x, & p_y to give us 3 simultaneous equations: ((label photons "d" for detected, and "u" for not))

$$E_d+E_u=E_{positronium}=m+E_+,\quad p_{d_y}+p_{u_y}=0=E_d-E_u\sin\theta,\quad p_{d_x}+p_{u_x}=p_{pos}=\mathrm{SQRT}(E_+^2-m^2)=E_u\cos\theta$$

where I have used the fact that $E_d=|\vec{p}_d|$ and $E_u=|\vec{p}_u|$ since these are photons, mass 0.

Looking back, we see that we have 3 equations and 3 unknowns, E_d, E_u & θ, so all that remains is to grind out the algebra and reach the expected solution. What more can I say?

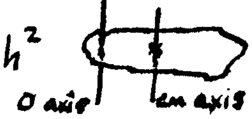
Other remarks: looking at last time's homework, I noticed that most people chose to solve the first problem by solving for γ and then grinding around...this works, of course, but using $E^2-p^2=m^2$ is much quicker & easier. One reason that that equation is so nice is that it defines an <u>invariant</u>, m. Rest mass m is the same, no matter what coordinate system you use, what velocity you're moving relative to the system, etc. E certainly depends on the observer's state of motion, and so does $\vec{p}$; by going to the center-of momentum frame of the system (if one exists), you make $\vec{p}_{tot}=0$, for instance. But m is independent of all that. The search for coordinate-invariant things tends, historically, to have been very productive --it was the philosophical motivation that lead Einstein to general relativity, for instance. The beauty & power of vectors is due to the fact that they let you make general, coordinate--free statements about things, like "$\vec{F}=\frac{d}{dt}\vec{p}$", true no matter how you happen to choose your x, y, and z axes. Coordinates are artificial, a human construction; they're very useful for doing particular calculations and for crunching numbers in a computer, but they don't exist in Nature, and the real physics of things <u>must</u> be the same no matter what coordinates you take. *Enough philosophy... I'll fill out this page with some good formulae that we've seen recently:*

From Newtonian mechanics:

	LINEAR		ANGULAR	
force	$\vec{F}=\frac{d\vec{p}}{dt}=m\vec{a}$ (for $\vec{p}=m\vec{v}$)	$\longleftrightarrow$	torque $\vec{M}=\frac{d\vec{L}}{dt}=\vec{r}\times\vec{F}=I\frac{d\vec{\omega}}{dt}$ (for $\vec{L}=I\vec{\omega}$)	(13-13) (14-12)
	$F_x=-\frac{dU}{dx}$	$\longleftrightarrow$	$M_z=-\frac{dU}{d\theta}$	(14-13)
	$K.E.=\frac{1}{2}mv^2$	$\longleftrightarrow$	$K.E.=\frac{1}{2}I\omega^2$	

$I=\sum m_iR_i^2=\int r^2dm=\int\rho r^2\,d(\text{Volume})\equiv mk^2$ (k is "radius of gyration) (14-25)?

$\parallel$ axis thm: $I_0=I_{cm}+mh^2$



(14-24a) | $\perp$ axis thm: $I_z=I_x+I_y$ for flat objects in x-y plane

$$\boxed{E=\gamma m_0c^2;\quad \vec{p}=\gamma m_0\vec{v};\quad E^2-p^2c^2=m_0^2c^4.}$$

Mark