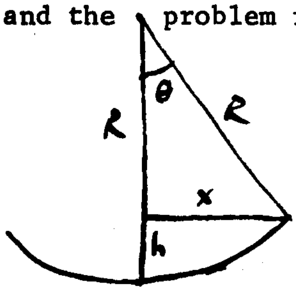
pH 1 section 7 newsletter--2 Feb 76--Mark Zimmermann, 133 Bridge, X 1665 & 103 MJ, 449-9203.

I was shocked at the lecture last Friday, until I realized that it was only supposed to be a preview of coming attractions, and wasn't supposed to be teaching anything much. The first 2 chapters of FVW are that way, certainly....

For the quiz: it's really all stuff we've done before. Part a), we can ask what the restoring force is when the mass is given a small displacement. If we're lucky, the force will be proportional to displacement with some constant of proportionality "k"...the law will look like $F=-kx$, and we can pretend that we just have a spring, of spring constant k. (If we're not lucky and F varies as some other power of x, we don't have simple harmonic motion and the problem is more difficult (or impossible) to solve.)



So, to do it: for a displacement angle θ, the mass moves $x=R\sin\theta$ horizontally and $h=R(1-\cos\theta)$ up; small θ allows us to approximate $\sin\theta=\theta$ and $\cos\theta=1-\theta^2/2$; so, we have $x=R\theta$, $h=\theta^2R/2$, and potential energy $U=mgh=mg\theta^2R/2=mgx^2/2R$, so restoring force $F=-dU/dx=-mgx/R$.

So, we've got a spring with constant $k=mg/R$. We know (or can derive from solving the differential equation $F=ma$ for a spring) that frequency of oscillation ω = SQRT(k/m)...so we've found answer ω_0=SQRT(g/R). This is only good for small oscillations, since we had to approximate sine and cosine for small θ...for larger oscillations, the motion is NOT simple harmonic (sine-wave), but messier...maybe elliptic functions, I don't know. Ask your math TA what those are!

Another way to find F(x) would be to find what slope the circle has as a function of x (that's actually what I was doing above, in disguise), resolve the gravitational force -mg(z) into components perpendicular and parallel to the circle, and take that parallel component as the restoring force. It will turn out to be about -mgx/R for small values of x....

Part b), since we know the motion is simple harmonic from the form of the restoring force, with frequency ω_0, we know that angle θ and position x will vary like $\sin\omega_0 t$ or $\cos\omega_0 t$ or some combination thereof, depending on initial conditions. For initial speed = 0, we can't have any sine part, since $d(\sin\alpha t)/dt = \alpha$ at $t=0$. At $t=0$, the cosine is 1, and to get the right amplitude we just have to multiply by θ_0 (or $x_0=R\theta_0$, if we want linear instead of angular displacements). [Recall, of course, that since the differential equation is LINEAR, any multiple of a solution, or any sum of solutions, is also a solution.] So, the answer seems to be $\theta(t)=\theta_0\cos(\omega_0 t)$, or the corresponding thing for x(t).

Comments on the homework assignment: it's all mathematics this time, some of it cute, but not terribly exciting. You should be able to do the stuff, in order to be able to solve problems. FVW Ch.2 no. 1 seems to be designed to force you to use the graphical method described in the text...you can do it with trig identities, but it gets very messy. Those pictures (I'll do some in class)haven't been that useful to <u>me</u> in calculating in the past, but it is nice to be able to visualize what's going on qualitatively. As for the LV problems --I'm not sure what you are supposed to assume in order to prove what is requested. It's easy but not inspirational to take the power series as the definition of sine, cosine, and exp...then algebra solves 18.3. Problem 18.B1 is trivial once you have seen it, or a variant of it, once, or if you are used to visualizing what multiplication does in the complex plane (problem 18.B3). The other part of 18.B3, proving angle-sum trig identities, is just a grind. I have some space left, so I'll fill it with mathematical data that's valuable to know. [Again, I remind you that I'd be happy if you would come by my office or room, any time, especially to talk about physics. If I'm trying to work, I can set up a time to meet when I'm less busy. Try extension 1665!]

$$(1+x)^n = 1 + n\frac{x^1}{1!} + n(n-1)\frac{x^2}{2!} + n(n-1)(n-2)\frac{x^3}{3!} + \dots$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + - \dots = \frac{e^{ix}+e^{-ix}}{2}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + - \dots = \frac{e^{ix}-e^{-ix}}{2i}$$

$$\boxed{e^{ix} = \cos x + i\sin x}$$

Mark