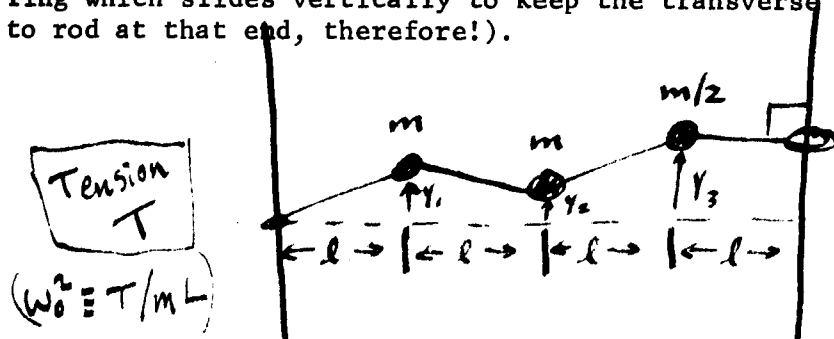


First, a variant on the last quiz. So many people misunderstood what physical system was being described, with those massless frictionless rings at each end, that I marked down a special symbol in my records & didn't really grade their quizzes. If you want to, consider the following problem: a string, length  $4L$ , with masses  $m$ ,  $m$ , and  $m/2$  along it. Tension in the string is  $T$ , the left end is fixed at  $y=0$ , the right end has a frictionless massless ring which slides vertically to keep the transverse force zero on it (string is perpendicular to rod at that end, therefore!).



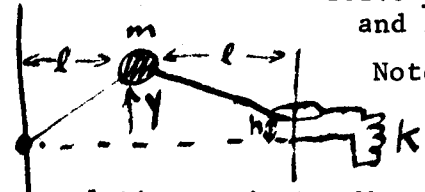
- (a) write the equations of motion (3 simultaneous diff. eqns.)
- (b) turn them into algebraic eqns, and solve for the frequencies of the 3 normal modes. [I got  $\omega^2/\omega_0^2 = 2, 2 \pm \sqrt{3}$  but I could be wrong!]

(c) sketch the shape of the string when in each of these normal modes, at some instant when all the masses are not at  $y=0$ . (You can solve for the normal modes exactly, if you want to...I haven't tried that yet myself...may be messy.)

I made the third mass  $m/2$  instead of  $m$  because I couldn't solve the cubic equation I got with all three masses equal to  $m$ . Try it that way, if you want to. The normal modes will be simpler, I suspect, when all masses= $m$ , but the frequencies won't be. Quiz above is open books & notes, but do LIST ALL REFERENCES USED, and if you run across this problem in some book, don't read any farther. No time limit, but don't spend more than an hour or two...it's not worth the effort. No consultation with others until after you're done working.

Another problem, for extra credit & fun: consider a system with one mass  $m$  on a string, fixed at left end (length  $L$  from mass) and at right (also  $L$  away) with a sliding ring--BUT with a spring attached to the ring that tends to pull it toward  $y=0$ , with spring constant  $k$ .

Solve this!! Write down the equation of motion for the mass, and find the frequency of harmonic motion!



Note: if the spring constant  $k$  goes to infinity, the right end is effectively fixed and we have the old homework problem that you've solved; if  $k \rightarrow 0$ , the end becomes free. So, there are two limits that you can check your

solution against. My picture isn't too clear; ask me about it and I'll try to describe it again.

Other news: I'll be going to Palomar to fool around, Saturday-Tuesday, and on Wednesday next there's a boat trip to Catalina for the SCUBA class...so, Rich Flammang will be taking over Mon/Wed at 3 for me. He's in the same office as I am, so if he forgets to show up, please try to remind him. Turn in the quiz & bonus question by Friday afternoon if you want it back early; I'll leave graded things in my TA mailbox.

Supplementary comments for you math freaks: a boundary condition which specifies the VALUE of a function at a boundary, as we do when we say that one end of the string is fixed at  $y=0$ , is called a "Dirichlet" b.c. If the b.c. specifies the derivative of the function at the boundary, as for our sliding rings (slope for string is zero), it's a "Neumann" b.c. Specifying BOTH value and derivative is "Cauchy" boundary condition.... The bonus problem is a mixed thing...the spring makes a connection between slope & value. Ask your math TA about this stuff, if you're interested...I don't know much more than the above.

Another comment: have you noticed that the eigenvalues of the matrices that we run across all are non-negative? We know what the zero-frequency normal modes correspond to; what would happen if one of the normal mode frequency-squared values became negative??

Another attempt to draw extra credit problem:



Good luck!  
-Mark